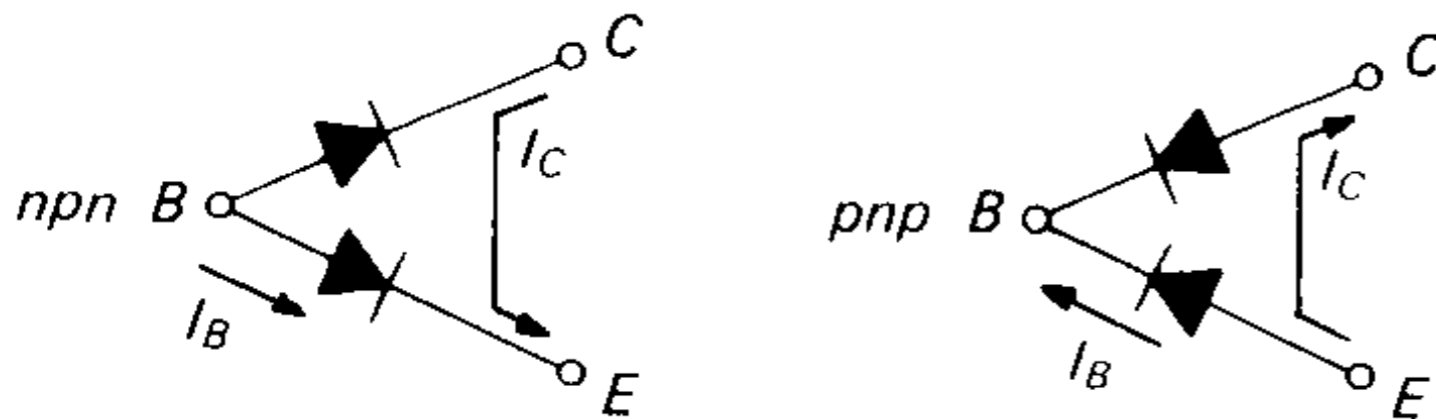
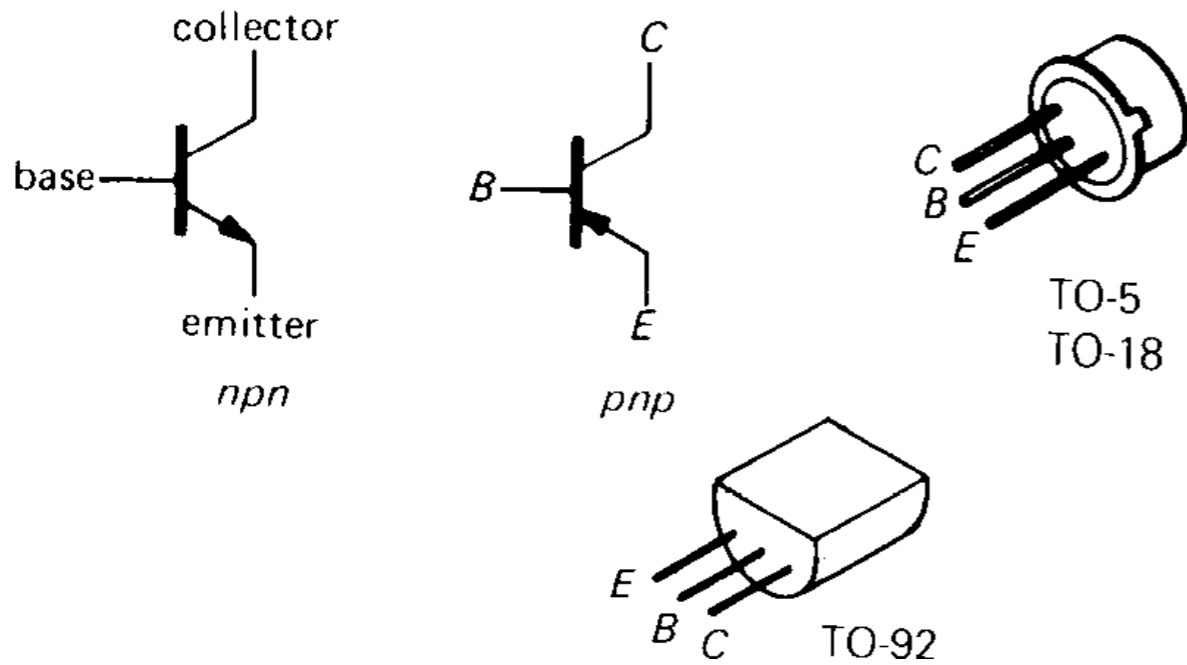


Transistor Models

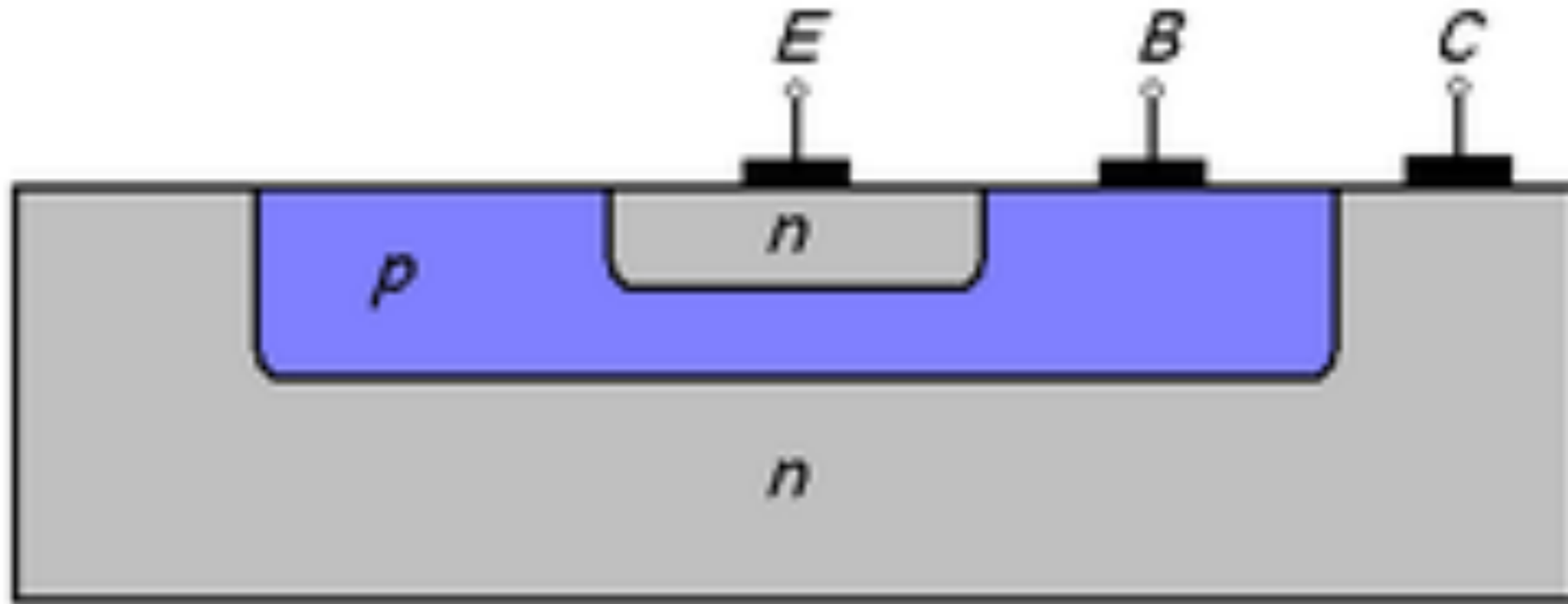
- Review of Transistor Fundamentals
- Simple Current Amplifier Model
 - Transistor Switch Example
 - Common Emitter Amplifier Example
- Transistor as a Transductance Device - Ebers-Moll Model
- Other Models: h-parameter, Gummel-Poon charge control model.

Transistor Review



- Transistor is two opposed diodes.
- Direction of diodes determines if it is PNP or NPN.
- N - electron doped region.
- P - hole doped region.

Transistor Review



- Under normal operation, when current flows between Base and Emitter (drift current in a field), electrons in the highly doped E region can diffuse across the (small) base region, and be swept across the reverse biased Collector region.

Current Amplifier Model

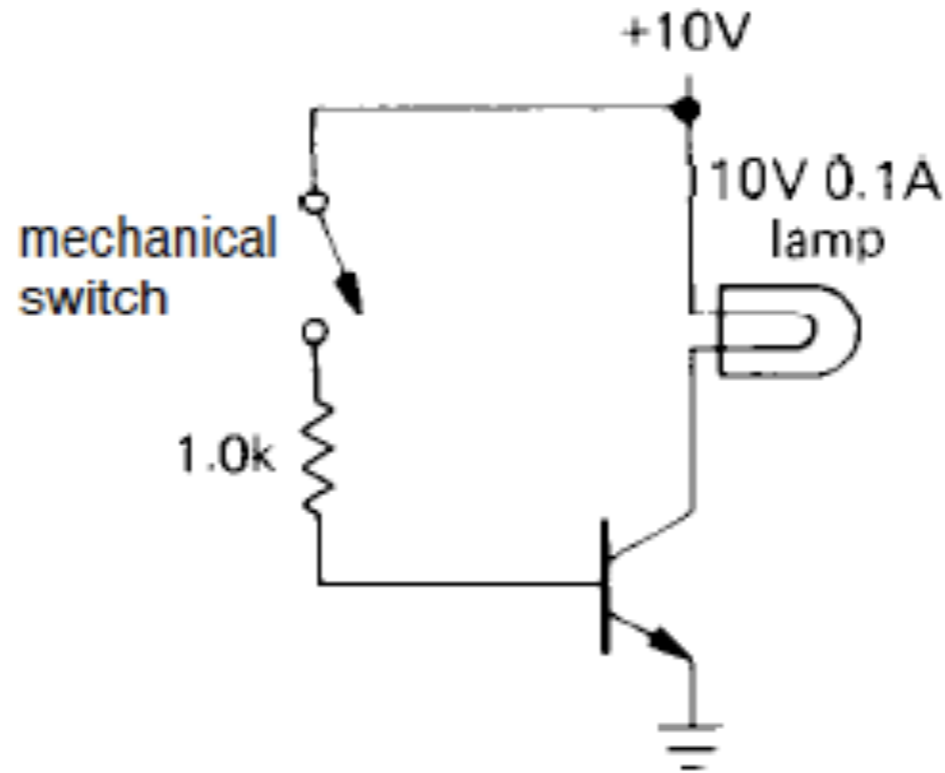
1. Collector must be more positive than emitter.
2. The Base-Emitter and Base-Collector junctions behave like diodes. Normally, the BE is forward biased, and the BC is reverse biased.
3. Transistors have maximum values of I_c , I_b and V_{ce} that cannot be exceeded. Exceeding these values results in a broken transistor.
4. When rules 1-3 are obeyed, the Collector Current is roughly given by:

$$I_c = h_{fe} I_b = I_b$$

Current Amplifier Model

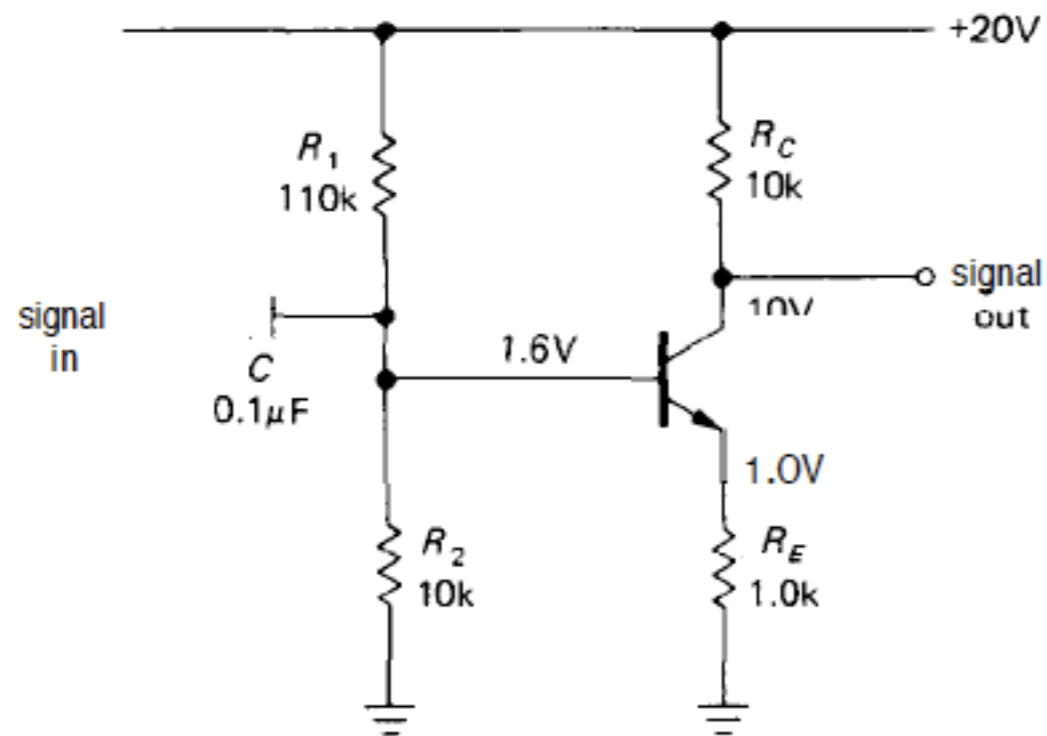
- h_{fe} is typically 100, but varies wildly with temperature, I_c , and between transistors.
- I_c is not due to diode conduction, it is due to diffusion of carriers from the heavily doped emitter region (which only occurs when the Base-Emitter region is forward biased).
- Some simple examples:
 - simple switch
 - Common-Emitter Amplifier

Simple Switch



- Control large currents with a small one.
- Switch open, no conduction (no Base-Emitter current)
- Switch closed, base voltage rises to 0.6 V (turn on voltage of diode). Current begins to flow through Collector-Emitter.
- 9.4 V produces, 9.4 mA into base, 940 mA across bulb, by rule 4. (not true, actually, 940 mA violates rule 1).

Common Emitter Amplifier



$$V_c = V_{cc} - I_c R_c$$

$$I_e = I_c + I_b$$

$$V_e \approx V_b$$

$$A_v = \frac{V_{out}}{V_{in}} \approx \frac{R_c}{R_e}$$

- Common Emitter Amplifier uses a current amplifier (BJT) to produce a voltage amplifier.
- Same derivation applies to circuit used in lab (not as straightforward, however).

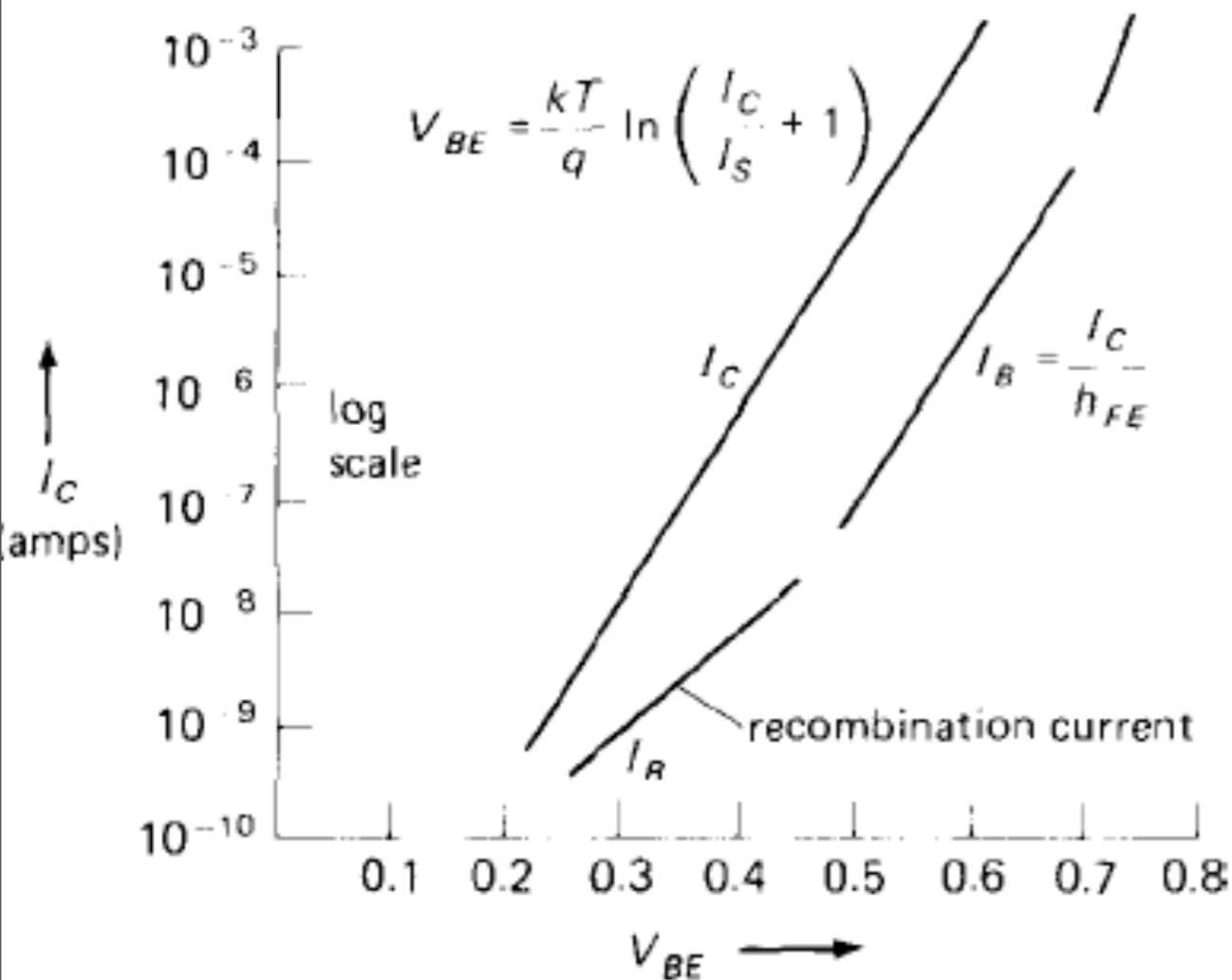
Ebers-Moll Model

- Transistors are not simple current amplification devices, better described as transductance devices.
- Transductance device: output current determined by input voltage: I_c is a function of V_{be} .

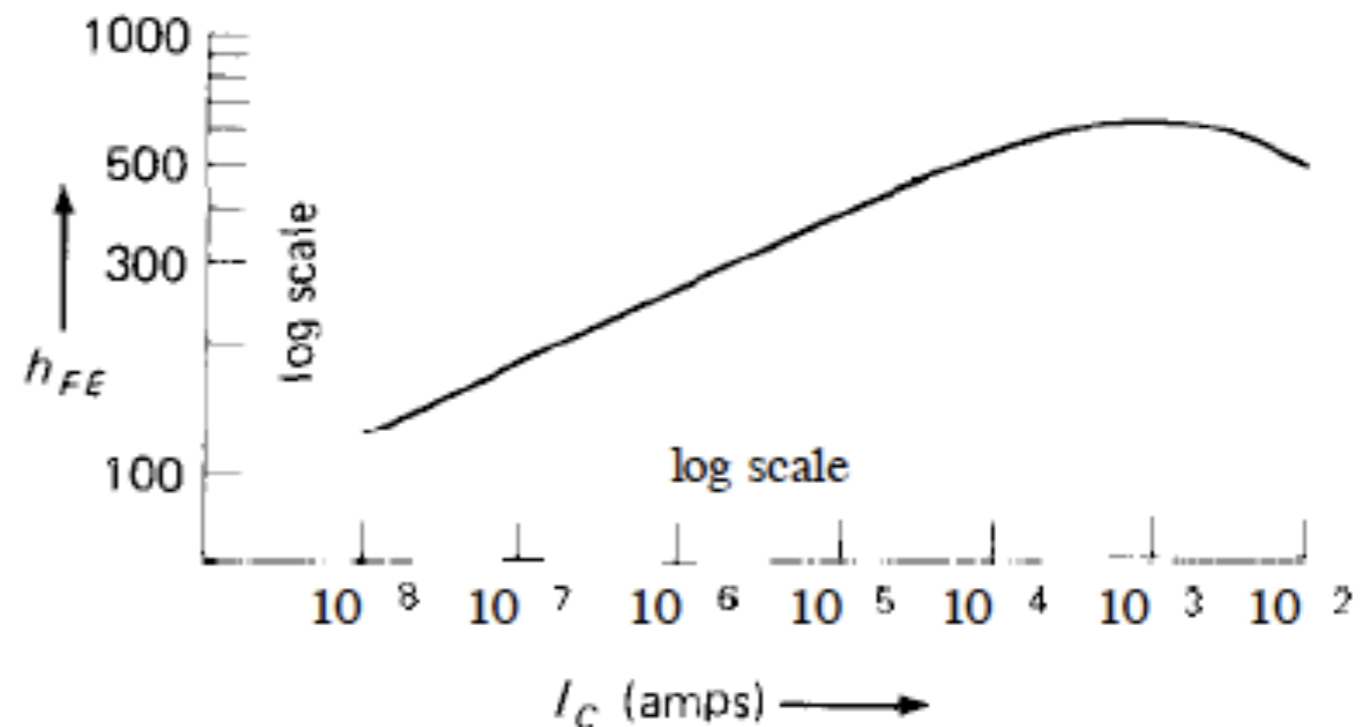
$$I_c = I_s \left(e^{\frac{V_{be}}{V_o}} - 1 \right)$$

- Look familiar?
- I_s is saturation current of the transistor (slightly different than I_o from diode equation).

Ebers-Moll Model



- I_C is a function of V_{be} , not I_b .



- h_{fe} is a bad parameter.

Other Models

- Many, many models of transistor action.
- h-parameter and hybrid-pi model are similar, designed for small signals, involve using an equivalent circuit to model transistor behaviour. Makes it easy to describe frequency dependence.
- Gunter-Poon Model, similar to EM model, but more complicated/more correct.
- Wikipedia is a good place to start.
- Datasheets are gold. (google “2n3904 datasheet”)

Chapter 5: Bipolar Junction Transistors



5.6. BJT circuit models

- [5.6.1. Small signal model \(hybrid pi model\)](#)
- [5.6.2. Large signal model \(Charge control model\)](#)
- [5.6.3. SPICE model](#)

A large variety of bipolar junction transistor models have been developed. One distinguishes between small signal and large signal models. We will discuss here first the hybrid pi model, a small signal model, which lends itself well to small signal design and analysis. The next model is the charge control model, which is particularly well suited to analyze the large-signal transient behavior of a bipolar transistor. And we conclude with the derivation of the SPICE model parameters.

5.6.1. Small signal model (hybrid pi model)



The hybrid pi model of a BJT is a small signal model, named after the “ π ”-like equivalent circuit for a bipolar junction transistor. The model is shown in Figure 5.6.1. It consists of an input impedance, r_{π} , an output impedance r_o , and a voltage controlled current source described by the transconductance, g_m . In addition it contains the base-emitter capacitances, the junction capacitance, $C_{j,BE}$, and the diffusion capacitance, $C_{d,BE}$ and the base-collector junction capacitance, $C_{j,BC}$, also referred to as the Miller capacitance.

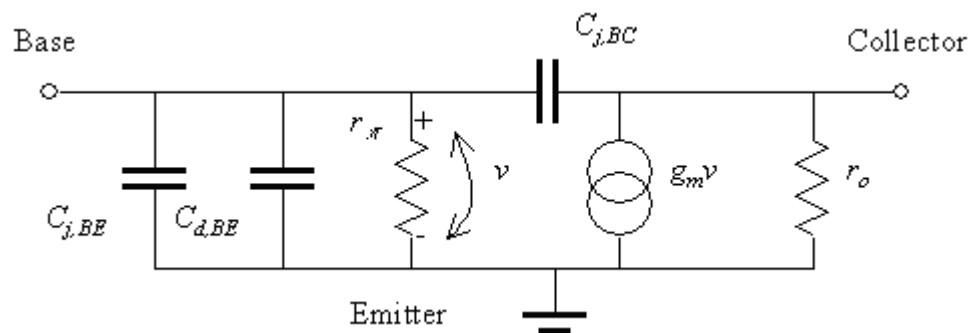


Figure 5.6.1. : Small signal model (hybrid pi model) of a bipolar junction transistor.

The transconductance, g_m , of a bipolar transistor is defined as the change in the collector current divided by change of the base-emitter voltage. Using (5.4.6) one obtains:

$$g_m = \frac{\Delta \partial I_C}{\partial V_{BE}} = \frac{I_C}{nV_t} \quad (5.6.1)$$

The base input resistance, r_{π} , is defined as the change of the emitter-base voltage divided by the change of base current.

$$r_{\pi} = \frac{\Delta \partial V_{BE}}{\partial I_B} = \beta \frac{\partial V_{BE}}{\partial I_C} = \frac{\beta}{g_m} = \frac{nV_t}{I_B} \quad (5.6.2)$$

The output resistance, r_o , is defined as:

$$r_o = \frac{\Delta \partial V_{CE}}{\partial I_C} \cong \frac{\partial V_{CE}}{\partial I_C} = \frac{|V_A|}{I_C} \quad (5.6.3)$$

The base-emitter and base-collector junction capacitances are given by:

$$C_{j,BE} = \frac{C_{j,BE0}}{\sqrt{1 - \frac{V_{BE}}{\phi_{i,BE}}}} \quad (5.6.4)$$

$$C_{j,BC} = \frac{C_{j,BC0}}{\sqrt{1 - \frac{V_{BC}}{\phi_{i,BC}}}} \quad (5.6.5)$$

for the case where the base-emitter and base-collector junctions are abrupt. Since the base-emitter is strong forward biased in the forward active mode of operation, one has to also include the diffusion capacitance of the base:

$$C_{d,BE} = \frac{I_E}{V_t} \tau_B \quad (5.6.6)$$

Based on the small signal model shown in Figure 5.6.1, we can now calculate the small signal current gain versus frequency, h_{fe} , of a BJT biased in the forward active mode and connected in a common emitter configuration. The maximum current gain is calculated while shorting the output, resulting in:

$$h_{fe} = \frac{i_C}{i_B} = \frac{\beta}{1 + j\omega(C_{j,BE} + C_{d,BE})\tau_\pi} \quad (5.6.7)$$

The unity gain frequency, f_T , also called the transit frequency is obtained by setting the small signal current gain, h_{fe} , equals to one, resulting in:

$$\left| \frac{i_C}{i_B} \right| = 1 \cong \frac{\beta}{2\pi f_T (C_{j,BE} + C_{d,BE})\tau_\pi} \quad (5.6.8)$$

This transit frequency can be expressed as a function of the transit time, τ :

$$f_T = \frac{1}{2\pi \tau} \quad (5.6.9)$$

Where the transit time, τ , equals:

$$\tau = \frac{C_{j,BE} nV_t}{I_E} + \frac{w_B^2}{2D_{n,B}} = \tau_E + \tau_B \quad (5.6.10)$$

The circuit model therefore includes the charging time of the base-emitter capacitance, τ_E , as well as the base transit time, τ_B , but not the transit time of the carriers through the base-collector depletion region, τ_C .

The collector transit time was presented in section 5.5.1. and described by equation (5.5.4), which is repeated here:

$$\tau_C = \frac{t_C}{2} = \frac{x_{d,BC}}{2v_{sat}} \quad (5.5.4)$$

The total transit time then becomes:

$$\tau = \frac{C_{j,BE} nV_t}{I_E} + \frac{w_B^2}{2D_{n,B}} + \frac{x_{d,BC}}{2v_{sat}} = \tau_E + \tau_B + \tau_C \quad (5.6.11)$$

The corresponding transit frequency, f_T , can still be calculated using (5.6.9).

While the unity gain frequency, f_T , is an important figure of merit of a bipolar transistor, another even more important figure of merit is the maximum oscillation frequency, f_{MAX} . This figure of merit predicts the unity power gain frequency and as a result indicates the maximum frequency at which useful power gain can be expected.

from a device. The maximum oscillation frequency, f_{MAX} , is linked to the transit frequency, f_T , and is obtained from:

$$f_{max} = \sqrt{\frac{f_T}{2\pi R_B C_{j,BC}}} \quad (5.6.12)$$

Where R_B is the total base resistance and $C_{j,BC}$ is the base-collector capacitance. The total base resistance consists of the series connection of metal-semiconductor contact resistance, the resistance between the base contact metal and the emitter and the intrinsic base resistance. Assuming a base contact, which is longer than the penetration depth this base resistance equals

$$R_B = \frac{\sqrt{R_{s,c} \rho_c}}{L_{s,E}} + R_{s,BE} \frac{\Delta L}{L_{s,E}} + \frac{1}{3} R_s \frac{W_{s,E}}{L_{s,E}} \quad (5.6.13)$$

for a one-sided base contact, where $R_{s,c}$, $R_{s,BE}$ and R_s are the sheet resistances under the base contact, between the base contact and the emitter and underneath the emitter respectively. $L_{s,E}$ is the emitter stripe length of the emitter, $W_{s,E}$ is the emitter stripe width of the emitter and ΔL is the alignment distance between base contact and emitter. For a double-sided base contact, the total base resistance equals

$$R_B = \frac{\sqrt{R_{s,c} \rho_c}}{2L_{s,E}} + R_{s,BE} \frac{\Delta L}{2L_{s,E}} + \frac{1}{12} R_s \frac{W_{s,E}}{L_{s,E}} \quad (5.6.14)$$

The base-collector capacitance equals:

$$C_{j,BC} = \epsilon_s \frac{A_C}{x_{d,BC}} \quad (5.6.15)$$

Where A_C is the base-collector area.

5.6.2. Large signal model (Charge control model)

The charge control model of a bipolar transistor is an extension of the charge control model of a p-n diode. Assuming the "short" diode model to be valid, one can express the device currents as a function of the charge in each region, divided by the corresponding transit or lifetime. In the general case one considers the forward bias charges as well as the reverse bias charges. This results in:

$$I_E = \frac{\Delta Q_{p,E}}{t_{r,E}} + \frac{\Delta Q_{n,B,f}}{t_{r,B}} - \frac{\Delta Q_{n,B,r}}{t_{r,B}} + \frac{\Delta Q_{n,B,r}}{\tau_{n,B}} \quad (5.6.16)$$

$$I_B = \frac{\Delta Q_{p,E}}{t_{r,E}} + \frac{\Delta Q_{n,B,f}}{\tau_{n,B}} + \frac{\Delta Q_{p,C}}{t_{r,C}} + \frac{\Delta Q_{n,B,r}}{\tau_{n,B}} \quad (5.6.17)$$

$$I_C = -\frac{\Delta Q_{p,C}}{t_{r,C}} - \frac{\Delta Q_{n,B,r}}{t_{r,B}} + \frac{\Delta Q_{n,B,f}}{t_{r,B}} - \frac{\Delta Q_{n,B,f}}{\tau_{n,B}} \quad (5.6.18)$$

Under forward active mode of operation, this model can be simplified since the reverse mode components can be ignored. A transient model can be obtained by adding the rate of change of the charges over time. To further simplify the model, we also ignore the minority carrier charge, $\Delta Q_{p,E}$, in the emitter. This results in the following equations:

$$I_E = \frac{\Delta Q_{n,B,f}}{t_{r,B}} + \frac{dQ_{n,B,f}}{dt} \quad (5.6.19)$$

$$(5.6.20)$$

$$I_B = \frac{\Delta Q_{n,B,f}}{\tau_{n,B}} + \frac{1}{\beta_F + 1} \frac{dQ_{n,B,f}}{dt}$$

$$I_C = \frac{\beta_F}{\beta_F + 1} \left[\Delta Q_{n,B,f} + \frac{dQ_{n,B,f}}{dt} \right] \quad (5.6.21)$$

As an example we now apply this charge control model to the abrupt switching of a bipolar transistor. Consider the circuit shown in Figure 5.3.2.(a). As one applies a positive voltage to the base, the base-emitter junction becomes forward biased so that the collector current will start to rise. The input is then connected to a negative supply voltage, V_R . This reverses the base current and the base-emitter junction capacitance is discharged.

After this transient, the transistor is eventually turned off and the collector current reduces back to zero. A full analysis would require solving the charge control model equations simultaneously, while adding the external circuit equations. Such an approach requires numeric simulation tools.

To simplify this analysis and provide insight, we now assume that the base current is constant before and after switching. This approximation is very good under forward bias since the base-emitter voltage is almost constant. Under reverse bias, the base current will vary as the base-emitter voltage varies, but conceivably one could design a circuit that does provide a constant reverse current.

The turn-on of the BJT consists of an initial delay time, $t_{d,1}$, during which the base-emitter junction capacitance is charged. This delay is followed by the increase of the collector current, quantified by the rise time, t_{rise} . The rise time is obtained by applying the charge control equation for the base current, while applying a base current I_{BB} with the voltage source V_{BB} :

$$I_{BB} = \frac{\Delta Q_{n,B,f}}{\tau_{n,B}} + \frac{1}{\beta_F + 1} \frac{d\Delta Q_{n,B,f}}{dt} \quad (5.6.22)$$

where:

$$I_{BB} = \frac{V_{BB} - V_{BE}}{R_B} \cong \frac{V_{BB} - 0.7}{R_B} \quad (\text{for a silicon BJT}) \quad (5.6.23)$$

This differential equation can be solved resulting in:

$$\Delta Q_{n,B,f} = I_{BB} \tau_B \left[1 - \exp\left(-\frac{t}{t_{r,B}}\right) \right] \quad (5.6.24)$$

If the device does not reach saturation, the charge reaches its steady state value with a time constant $t_{r,B}$, which equals the base transit time of the BJT. The corresponding collector current will be proportional to the excess minority carrier charge until the device reaches saturation or:

$$I_C = \frac{I_{BB} \tau_B}{t_{r,B}} \left[1 - \exp\left(-\frac{t}{t_{r,B}}\right) \right] \quad \text{for } \Delta Q_{n,B,f} \leq \frac{V_{CC} - V_{sat}}{R_L} t_{r,B}$$

$$I_C = \frac{V_{CC} - V_{sat}}{R_L} \quad \text{for } \Delta Q_{n,B,f} \geq \frac{V_{CC} - V_{sat}}{R_L} t_{r,B} \quad (5.6.25)$$

A larger base voltage, V_{BB} , will therefore result in a larger charging current, I_{BB} , which in turn decreases the time and causes the BJT to saturate more quickly. There also will be more excess minority carrier charge stored in the base region after the BJT is turned on. The rise time, t_{rise} , is then obtained by finding the time when the saturation current is reached or:

$$t_{rise} = t_{r,B} \ln\left(1 - \frac{V_{CC} - V_{sat}}{R_L I_{BB} \tau_{n,B}} t_{r,B}\right) \quad (5.6.26)$$

While switching back to the negative power supply, V_R , the base current is reversed. As long as significant charge is still stored in the base region, the collector current will continue to exist. Only after this excess charge

is removed, will the base-emitter junction capacitor be discharged and the BJT be turned off. The removal of excess charge can take a significant delay time labeled as $t_{d,2}$ on the figure. Again we can calculate the time evolution of the excess charge and calculate the collector current from it. To first order the delay time, $t_{d,2}$, equals:

$$t_{d2} = \frac{I_{BB} \tau_{n,B}}{I_R} \quad \text{with } I_R \cong \frac{V_R}{R_B} \quad (5.6.27)$$

This delay time can be significantly larger than the rise time t_{rise} . Also note that a higher base turn-on current I_{BB} results in a larger turn-off delay as more minority carrier charge is stored in the base.

The actual fall time, t_f , depends on the remaining storage charge at the onset of saturation as well as the charge stored by the base-emitter junction capacitance.

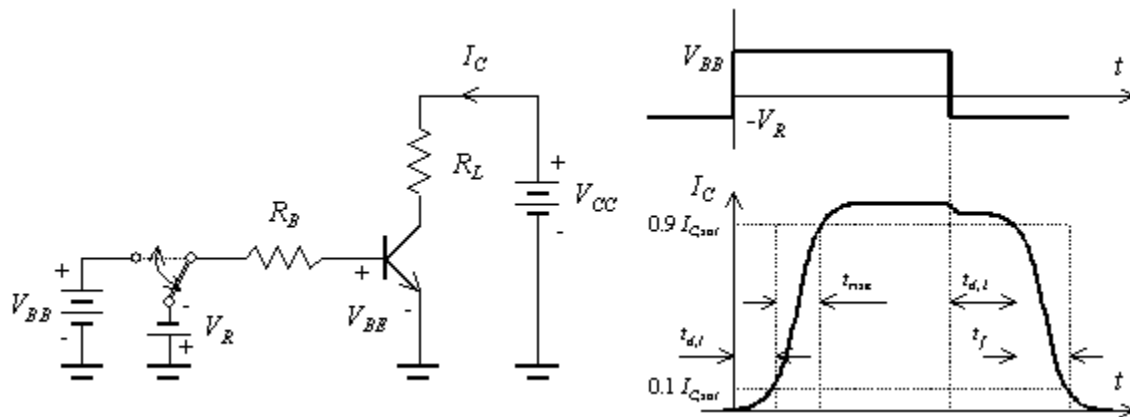


Figure 5.6.2. : Switching behavior of a BJT: a) bias circuit used to explain the switching behavior. b) Applied voltage and resulting collector current.

5.6.3. SPICE model



The SPICE model of a bipolar transistor includes a variety of parasitic circuit elements and some process related parameters in addition to the elements previously discussed in this chapter. The syntax of a bipolar transistor incorporates the parameters a circuit designer can change as shown below:

BJT syntax

Q []

+ []

.MODEL NPN(BF= BR= IS= CJE=

+ CJC= VJE= VJC= VAF= VAR=

+ NF= NR=)

BJT Parameters

BF Forward active current gain

BR Reverse active current gain

IS Transport saturation current

CJE Base-emitter zero-bias junction capacitance

CJC Base-collector zero-bias Junction capacitance

VJE Base-emitter built-in potential

VJC Base-collector built-in potential

VAF Forward mode Early voltage

VAR Reverse mode Early voltage

NF Forward mode ideality factor

NR Reverse mode ideality factor

Example:

Q1 3 2 1 BJTNAME

.MODEL BJTNAME NPN(BF=100 CJC=20pf CJE=20pf IS=1E-16)

where Q1 is one specific transistor in the circuit, while the transistor model "BJTNAME" uses the built-in model NPN to specify the process and technology related parameters of the BJT. The built-in model PNP is used for n-p bipolar transistors. A list of SPICE parameters and their relation to the parameters discussed in this text is provided in the table below.

SPICE variable	Equation
BF	$BF = \beta_F$ (see section 5.3.1)
BR	$BR = \beta_R$ (see section 5.3.2)
IS	$I_{s,n} = qn_i^2 A_E \left(\frac{D_{n,B}}{N_B w_B'} \right)$
CJE	$C_{j0,BE} = \sqrt{\frac{\epsilon_s q}{2 A_{,BE}} \frac{N_B N_E}{N_B + N_E}}$
CJC	$C_{j0,BC} = \sqrt{\frac{\epsilon_s q}{2 A_{,BC}} \frac{N_B N_C}{N_B + N_C}}$
VJE	$A_{,BE} = V_t \ln \frac{N_E N_B}{n_i^2}$
VJC	$A_{,BC} = V_t \ln \frac{N_C N_B}{n_i^2}$
VAF	$ V_{A,F} = \frac{q N_B w_B'}{C_{j,BC}}$
VAR	$ V_{A,R} = \frac{q N_B w_B'}{C_{j,BE}}$
NF	$n_F \cong 1 + \frac{V_t}{ V_{A,F} } \frac{C_{j,BE}}{C_{j,BC}}$
NR	$n_R \cong 1 + \frac{V_t}{ V_{A,R} } \frac{C_{j,BC}}{C_{j,BE}}$

Table 5.6.1. : Selected SPICE parameters of a BJT.

In addition, there are several parameters, which can be specified to further enhance the accuracy of the model.

such as:

RB base resistance

RE emitter resistance

RC collector resistance

MJE base-emitter capacitance exponent

MJC base-collector capacitance exponent

EG energy gap for temperature effect on IS

The exponents MJE and MJC are used to calculate the voltage dependence of the base-emitter and base-collector junction capacitances using:

$$C_{j,EE} = \frac{C_{j,EE0}}{\left(1 - \frac{V_{BE}}{A_{,BE}}\right)^m}, \text{ with } m = \text{MJE} \quad (5.6.28)$$

$$C_{j,BC} = \frac{C_{j,BC0}}{\left(1 - \frac{V_{BC}}{A_{,BC}}\right)^m}, \text{ with } m = \text{MJC} \quad (5.6.29)$$

This exponent allows the choice between a uniformly doped junction ($m = 1/2$), a linearly graded junction ($m = 1/3$) or an arbitrarily graded junction for which the exponent must be independently determined.

The temperature dependence of the transport saturation current is calculated from the energy bandgap, since the primary temperature dependence is due to the temperature dependence of the intrinsic carrier density, which results in:

$$I_{s,n} = qA_E \left(\frac{D_{n,B}}{N_B W_B} \right) \sqrt{N_c N_v} \exp \frac{-E_g}{kT} \quad (5.6.30)$$

The corresponding equivalent circuit is provided in Figure 5.6.3. The output resistance, r_o , was added to represent the Early effect, which is included in the BJT model by specifying VAF and VAR.

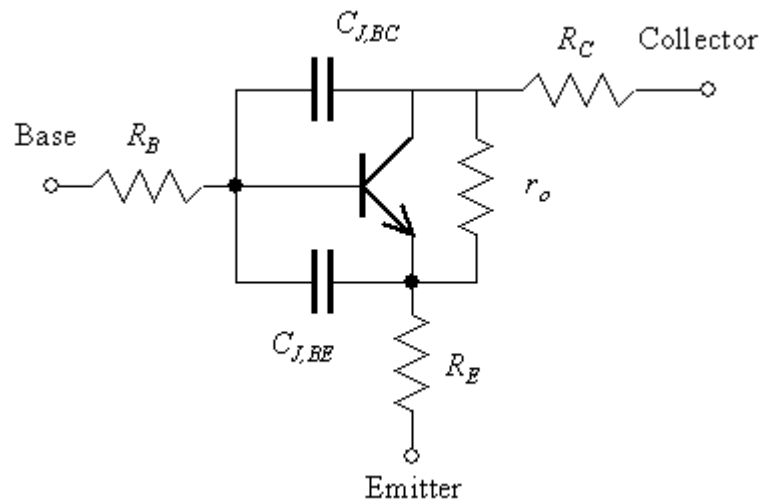


Figure 5.6.3. : Large signal model of a BJT including the junction capacitances.

Boulder, December 2

