Waves and Polarization in General

Wave means a disturbance in a "medium" that travels.

For light, the "medium" is the electromagnetic field, which can exist in vacuum.

The "travel" part defines a direction.

The "disturbance" can also have a direction (or none).

The direction of travel and the direction of disturbance don't have to be the same.

For example, consider a "Slinky" spring.

If I have a Slinky stretched out in the z direction, I can launch waves down it by wiggling it in the x direction.
I can also wiggle it in the y direction.

The wave is travelling in z, but the disturbance is in the x or y direction.

I could also wiggle the end of the spring in the z direction.
That will make "density" waves travel down the spring.

These different wiggles are different polarizations. There are two transverse polarizations and one longitudinal polarization.

Waves and Polarization (2)

Of course, I could wiggle the Slinky, or a rope, stretched in the z direction, in a plane at <u>any</u> angle transverse to the z direction, and still generate a wave that would travel down the rope. I'm not limited to x and y directions.

The transverse polarization is a <u>vector</u>, it has <u>direction</u> as well as <u>magnitude</u>.

I can represent that vector by its x and y components.

Of course, I could have chosen a <u>different coordinate system</u>, with x′ and y′ axes that are rotated relative to the x and y axes (but with the same z axis).

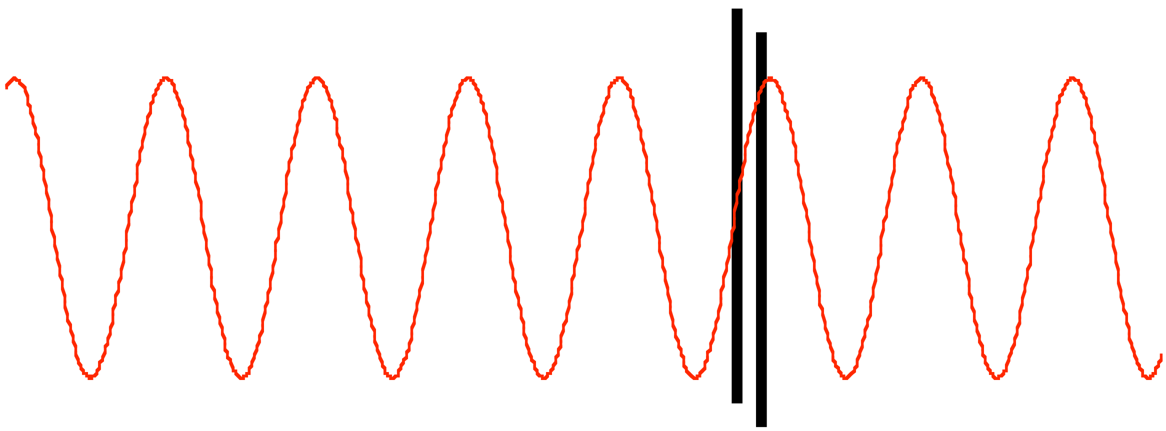
Then I would write the vector with numbers for its x′ and y′ components that are different from the numbers for the x and y components, even though they <u>represent</u> the same <u>physical</u> vector.

This concept that we can decompose the polarization of the same <u>physical</u> wave into <u>different components</u> when we use <u>different coordinate systems</u> is essential to understand what happens with polarizing filters.

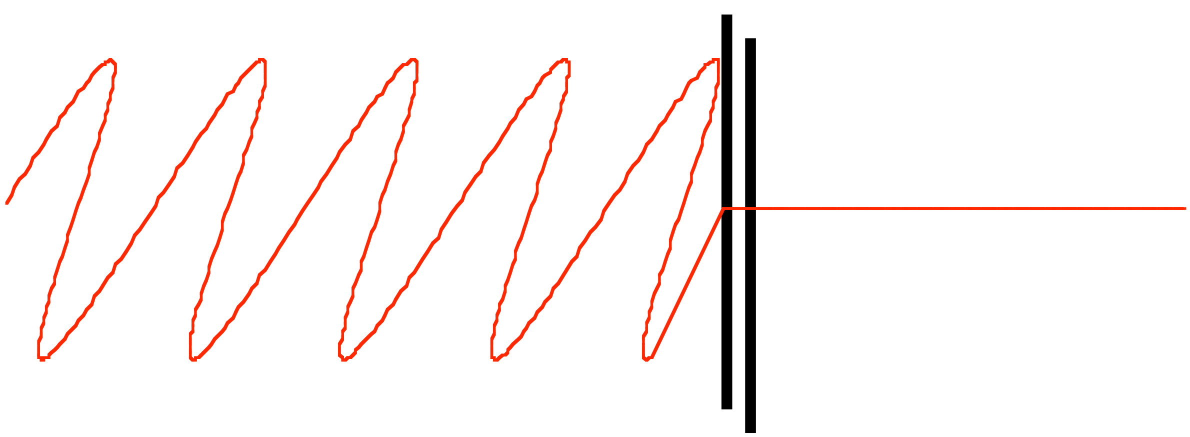
Polarizing Filters

Imagine I have a rope to launch waves down, and at some distance away the rope goes through a frictionless slot just barely big enough for the rope.

If I wiggle the rope <u>parallel</u> to the slot, the rope is free to slide in the slot, and the wave gets through.

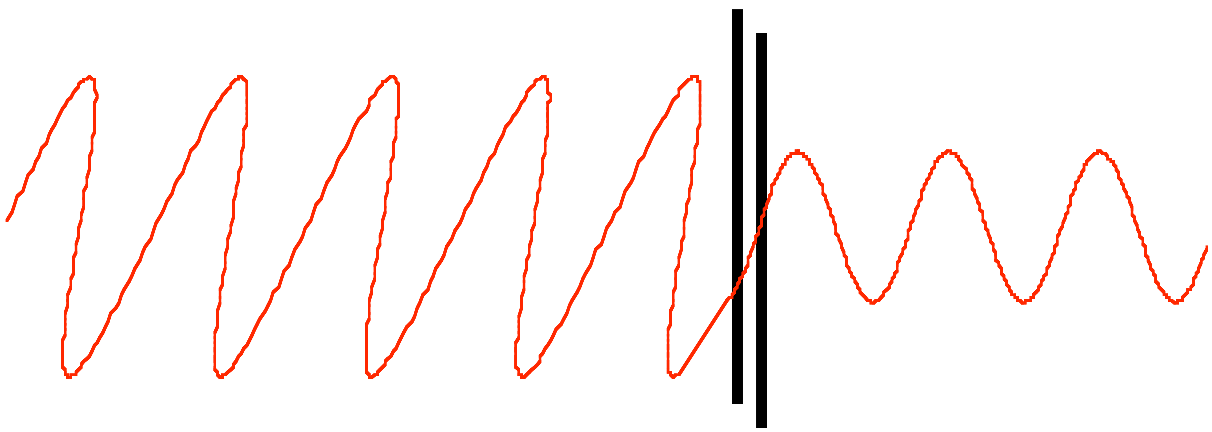


If I wiggle the rope <u>perpendicular</u> to the slot, the rope can't wiggle in the slot, so the wave doesn't get through (it will reflect back to me or get absorbed).



Polarizing Filters (2)

If I wiggle the rope at 45°, what happens? The rope slides in the direction that the slot allows, so a wave does come out the other side, but at lower amplitude



Also the "polarization" is <u>parallel to the slot</u> rather than to the initial 45° direction.

The same thing happens with light and polarizing filters.

The amount of light that gets through the filter depends on how parallel the polarization of the light is to the filter.

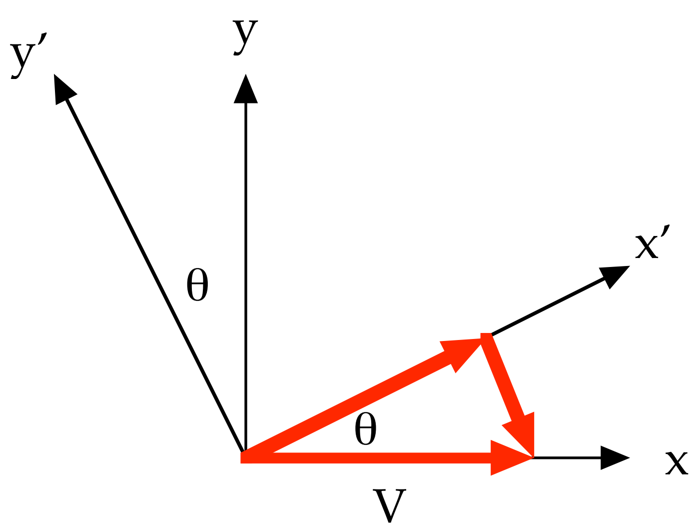
If the polarization is parallel, it nearly all gets through.
If the polarization is at right angles, none gets through.

But what about the general case, where the angle between the light polarization and the filter is arbitrary?

Polarizing Filters (3)

To get a quantitative answer, the trick is to express the incoming polarization vector in a coordinate system lined up with the axis of the filter.

If a vector points in the x direction in one coordinate system, and another coordinate system is rotated around the z axis by angle θ, what are the components of the same physical vector in the new coordinate system?



The x′ component in the rotated system is

$$x' = \left|\vec{V}\right| \cos\theta = x\cos\theta$$

The y′ component in the rotated system is

$$y' = -\left|\vec{V}\right| \sin\theta = -x\sin\theta$$

Polarizing Filters (4)

The electric field component in the wave field that is parallel to the filter axis is transmitted, and the electric field component that is perpendicular to the filter axis is not transmitted.

The transmitted electric field strength is just $\cos\theta$ times the original field strength, where θ is the angle between the original field direction and the filter axis.

Now $\cos\theta$ is a function that can be positive or negative, and that's sensible when we are talking about electric field vectors. The electric field that is positive in one coordinate system can be negative in another coordinate system.

But the intensity of light can't be negative!

The intensity (power) of light turns out to depend on the square of the electric field.

So the light intensity getting through the filter is proportional to $\cos^2\theta$. This is often called Malus' Law.

Two Polarizing Filters

If I start with unpolarized light, what intensity will get through an ideal polarizing filter

The value of $\cos^2\theta$, averaged over all θ, is $1/2$, so half the light will get through (with a real filter, it will be somewhat less than half, because there is some absorption of the "right" polarization too).

What if I have a second filter downstream of the first?

Remember, the light that gets through the first filter is completely polarized.

If the second filter is parallel to the first, whatever gets through the first slot will get through the second filter.

If the second filter is at right angles to the first filter, nothing that gets through the first filter will get through the second.

If the angle is in between, a fraction will get through.
The fraction will be in fact $\cos^2\theta$.

What happens if I have two filters, crossed at right angles so no light gets through, and I put another filter between?

Electromagnetic Waves and Polarization

Light is an electromagnetic wave. The electric and magnetic fields of the wave are always at right angles to the direction of propagation, and to each other. The direction of the electric field is the polarization direction.

One particular wave solution to Maxwell's Equations is

$$\vec{E} = E_0 \hat{x} \cos(kz - \omega t) \quad \vec{B} = \frac{E_0}{c} \hat{y} \cos(kz - \omega t) \quad \frac{\omega}{k} = c$$

The solution above is <u>linearly polarized in the x direction</u>. The convention is that the polarization direction is the direction of the electric field.

A wave moving in the z direction <u>linearly polarized in the y direction</u> would look like

$$\vec{E} = E_0 \hat{y} \cos(kz - \omega t) \quad \vec{B} = -\frac{E_0}{c} \hat{x} \cos(kz - \omega t) \quad \frac{\omega}{k} = c$$

We can superpose both of these fields to get a wave <u>polarized at +45°</u>:

$$\vec{E} = E_0 (\hat{x} + \hat{y}) \cos(kz - \omega t) \quad \vec{B} = \frac{E_0}{c} (\hat{y} - \hat{x}) \cos(kz - \omega t)$$

A wave <u>polarized at -45°</u> is physically totally different but looks very similar mathematically:

$$\vec{E} = E_0 (\hat{x} - \hat{y}) \cos(kz - \omega t) \quad \vec{B} = \frac{E_0}{c} (\hat{y} + \hat{x}) \cos(kz - \omega t)$$

Polarization and Speed of Light

For vacuum, or air, or water, or glass, the speed of light is the same for any polarization (although it may depend on frequency or wavelength).

If a material has some internal structure that makes the electrical properties different in some directions, the speed of light can be different for different polarizations. This is called dichroism.

Many crystals are like this. Quartz is an example. It has the same chemical makeup as glass (silicon dioxide), but glass has random bond directions and orientation, while quartz is very regular. If the polarization of light is parallel to some directions, the speed of light is different.

Materials that are stretched can also have this property. For instance, transparency plastic is stretched during manufacturing, so the speed of light is different for polarization parallel and at right angles to the stretch axis.

Watch what happens when I put this transparency between two crossed polarizers!

Polarization and Speed of Light (2)

The stretching means the fast and slow polarization axes are parallel and perpendicular to the vertical axis, 90 degrees apart.

When the transparency is vertical or horizontal, the light that gets through the first polarizer is aligned with either the fast or slow axis, and propagates at whatever speed that implies. The light gets blocked by the second polarizer.

When the transparency is at 45 degrees, we can decompose it into components parallel to the fast axis, and parallel to the slow axis.

The two components will have equal amplitudes. Initially, they are in phase with each other. But they travel at different speeds, so they get out of phase.

Even though the amplitudes of the two 45-degree components are still the same, because they got out of phase, they no longer add up to the original light with the original polarization.

Since some of the light has the "wrong" polarization, it gets through the second polarizer.

Circular Polarization

Going back to our stretched rope analogy, we aren't limited to shaking the end of the rope back and forth in a single plane. We could also move the end in a circle.

That would produce a "helical" wave travelling down the stretched rope. The motion would not be confined to a single plane.

This is circular polarization (the kind of polarization we have been talking about before is linear polarization).

The motion of the end of the rope would be described as

$$x = R\cos(\pm 2\pi ft) = R\cos(\pm\omega t) = R\cos(\omega t)$$
$$y = R\sin(\pm 2\pi ft) = R\sin(\pm\omega t) = \pm R\sin(\omega t)$$

The frequency in cycles per second is f, the frequency in radians per second is ω. The plus-minus signs are because we could be going clockwise or counterclockwise.

The disturbance would travel down the rope in the z direction as waves. The displacement at other points is

$$x(z) = R\cos(kz - \omega t)$$
$$y(z) = \pm R\sin(kz - \omega t)$$

Basically, we have waves in both x and y, out of phase in time and space.

Circular Polarization of Light

We can also make circularly-polarized light. To do that, we add x and y linear polarizations, but we also make them out of phase in time.

A right-circular polarized wave looks like

$$\vec{E} = E_0\left[\hat{x}\cos(kz - \omega t) + \hat{y}\sin(kz - \omega t)\right]$$

$$\vec{B} = \frac{E_0}{c}\left[\hat{y}\cos(kz - \omega t) - \hat{x}\sin(kz - \omega t)\right]$$

The electric field at a given point in space rotates in direction at constant magnitude. The magnetic field rotates the same direction but is always 90° away.

A left-circular wave looks like

$$\vec{E} = E_0\left[\hat{x}\cos(kz - \omega t) - \hat{y}\sin(kz - \omega t)\right]$$

$$\vec{B} = \frac{E_0}{c}\left[\hat{y}\cos(kz - \omega t) + \hat{x}\sin(kz - \omega t)\right]$$

This sounds pretty delicate, but some systems do it naturally (some microwave antennas, some atomic transitions).

Linear To Circular Filter

We can make a device that will turn linearly polarized light to circular polarized light and back again.

Get a dichroic material with fast and slow axes that are 90 degrees apart (in direction).

Arrange it so the fast and slow axes are at +/- 45 degrees relative to the linear polarization.

Make the layer just the right thickness such that the fast and slow waves will be 90 degrees out of phase in time and space.

The light that comes out will be circularly polarized.

What happens if we make the layer thick enough that the slow wave is exactly 360 degrees out of phase with the fast wave?

What happens if we make the layer thick enough that the slow wave is exactly 270 degrees out of phase with the fast wave?

What happens if we make the layer thick enough that the slow wave is exactly 180 degrees out of phase with the fast wave?

Circular to Linear Polarization

If we add right-circular and left-circular light, we get

$$\vec{E} = E_0 \begin{bmatrix} \hat{x}\cos(kz - \omega t) + \hat{y}\sin(kz - \omega t) + \\ \hat{x}\cos(kz - \omega t) - \hat{y}\sin(kz - \omega t) \end{bmatrix}$$

$$\vec{B} = \frac{E_0}{c} \begin{bmatrix} \hat{y}\cos(kz - \omega t) - \hat{x}\sin(kz - \omega t) + \\ \hat{y}\cos(kz - \omega t) + \hat{x}\sin(kz - \omega t) \end{bmatrix}$$

This obviously reduces to

$$\vec{E} = E_0 \left[2\hat{x}\cos(kz - \omega t) + 0 \right]$$

$$\vec{B} = \frac{E_0}{c} \left[2\hat{y}\cos(kz - \omega t) + 0 \right]$$

which is just <u>linearly polarized in the x direction</u>.

If we subtract left-circular from right-circular, we get

$$\vec{E} = E_0 \left[0 + 2\hat{y}\sin(kz - \omega t) \right]$$

$$\vec{B} = \frac{E_0}{c} \left[0 - 2\hat{x}\sin(kz - \omega t) \right]$$

which is <u>linearly</u> polarized in the <u>y direction</u> (and out of phase with the original, but normally you don't see the phase of light)

Circular Dichroism (Optical Activity)

There are some materials where the speed of light is different for left and right hand circular polarizations.

This is called optical activity or circular dichroism.

These materials will rotate the polarization of linearly polarized light.

For this to happen, the material has to have a helical structure of its own that is left or right handed.

Many molecules come in left-handed and right-handed forms. It is not necessary for the material to be crystalline, even a liquid can show the effect.

Normal chemistry will produce both forms in equal amounts and the effect will cancel.

But biological processes based on enzymes often make only one form or the other, so optical activity is common. Sugars are a good example.

Faraday Effect and Circular Polarization

The Faraday Effect is due to the fact that magnetic fields can cause the speed of light to be different for left-circular and right-circular polarized light in some materials.

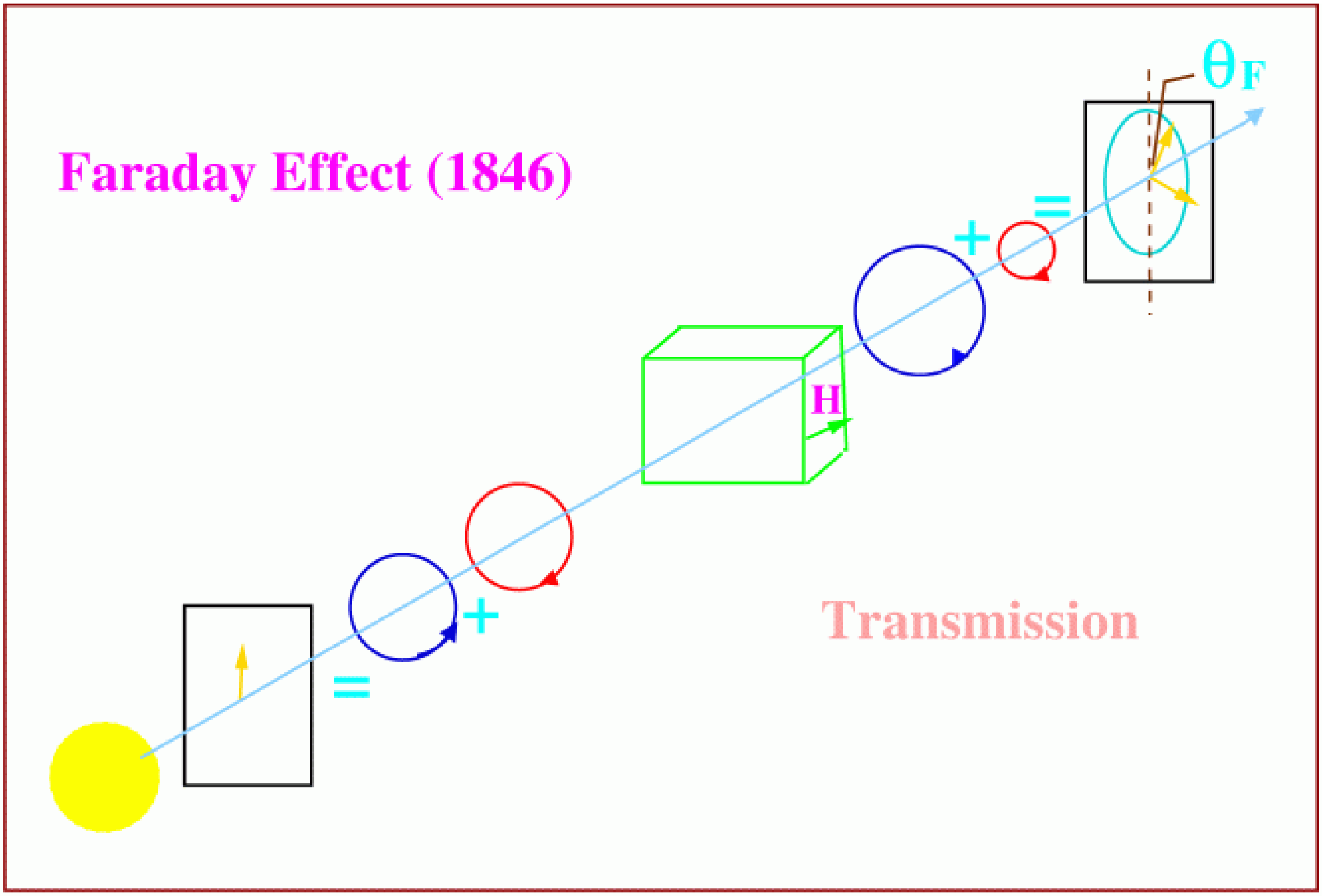
The electrons in the material want to orbit the field lines in one direction and not the other due to the $\vec{v} \times \vec{B}$ Lorentz force.

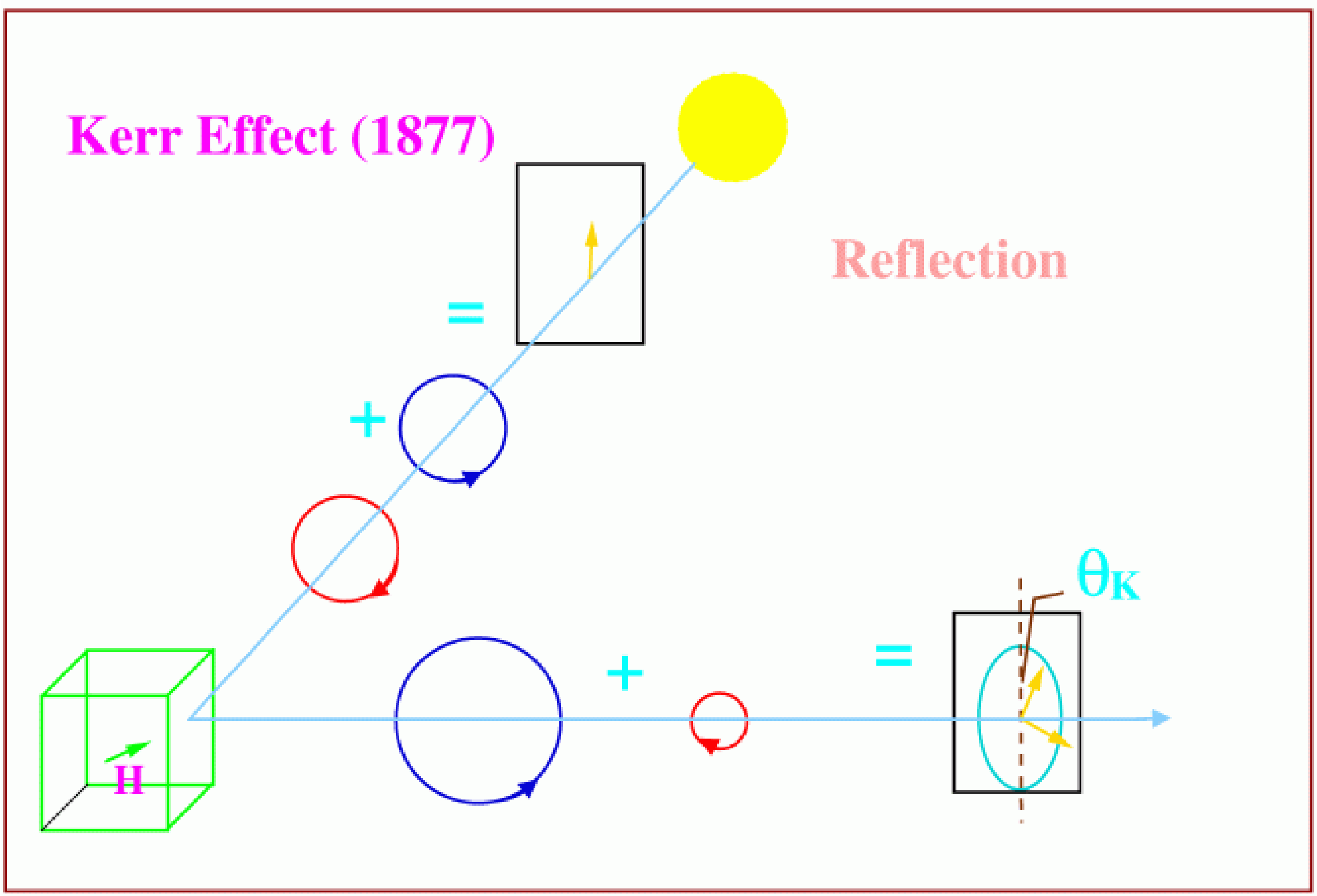
One circular polarization makes the electrons go the “right” way, the other makes them go the “wrong” way, so the “drag” the electrons exert on the light is different for the two circular polarizations.

We can write the original linear polarization as an equal mix of left and right circular polarizations. The left and right polarizations get out of phase as they travel through the material.

When we add the two circular polarizations up again, the relative phase shift causes the sum to be linearly polarized light with a different polarization direction.

The higher the magnetic field, the larger the velocity difference, so the larger the polarization rotation.


Faraday Effect (1846)
Transmission
H
θF


Kerr Effect (1877)
Reflection
H
θK

Circular Polarization and Complex Exponentials

With circular polarization, we are constantly dealing with x and y directions, and with phases. Complex exponentials can make the mathematical book-keeping easier.

First recall that

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad\qquad i^2 = -1$$

Now consider the following expression

$$\begin{aligned}&(\hat{x} \pm i\hat{y})e^{i\theta} = (\hat{x} \pm i\hat{y})(\cos\theta + i\sin\theta)\\ &= [\hat{x}\cos\theta \mp \hat{y}\sin\theta] + i[\hat{x}\sin\theta \pm \hat{y}\cos\theta]\end{aligned}$$

The real part of this looks similar to circular polarization. (the imaginary part is to, with a 90-degree phase shift)

Just like $\cos(kz - \omega t)$ is the real part of $\exp[i(kz - \omega t)]$, left and right-circular polarizations are the real part of

$$(\hat{x} \pm i\hat{y})\exp[i(kz - \omega t)]$$

Circular Polarization and Complex Exponentials (2)

If we add a phase ϕ to <u>right-circular</u> light, we get

$$(\hat{x} - i\hat{y})\exp\left[i(kz - \omega t + \phi)\right] = (\hat{x} - i\hat{y})\exp\left[i(kz - \omega t)\right]e^{i\phi}$$

Add the <u>opposite</u> phase-shift to <u>left-circular</u> light

$$(\hat{x} + i\hat{y})\exp\left[i(kz - \omega t)\right]e^{-i\phi}$$

then add these together we get

$$\left[\hat{x}\left(e^{i\phi} + e^{-i\phi}\right) - i\hat{y}\left(e^{i\phi} - e^{-i\phi}\right)\right]\exp\left[i(kz - \omega t)\right]$$
$$= \left[2\hat{x}\cos\phi + 2\hat{y}\sin\phi\right]\exp\left[i(kz - \omega t)\right]$$

Now if we take the real part of this, we get

$$\left[2\hat{x}\cos\phi + 2\hat{y}\sin\phi\right]\cos(kz - \omega t)$$

So if we add left and right circular polarizations with a phase shifts of $+\phi$ and $-\phi$, we rotate the linear polarization by angle 2ϕ.